Analytical development of Orthotropic plate on Winkler elastic foundation subjected to moving distributed load

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Abstract

In this present paper, analytical solutions of the fourth order partial differential equations describing the motion of rectangular orthotropic plate are obtained. An approach involving separation of variables, the Strubles’s asymptotic technique, convolution theory and the application of Laplace transform is developed. The conditions under which the plate will experience resonance phenomenon have been established for the orthotropic plate on constant elastic foundation under moving distributed masses.

Keywords: Orthotropic plate; Winkler elastic foundation; Load

1. Introduction

Rectangular plates on elastic foundation find use in many engineering structures and other areas of practical interest. The consideration of the effect such as elastic foundations on their behavior is among one of the most important research fields in mathematical physics and construction engineering and this has attracted the attention of many authors in this area.

There have many publications on the effect of the elastic foundation on the dynamic deflection and the response to loading condition to describe the behavior of the plate flexure (Timoshenko and Winowsky-Krieger 1959, Fryba 1995, Utku et al. 2000, Kobayashi and Sonoda 1989, Filonenko-Borodich 1940). The problem on elastic foundation becomes more complicated when the plate is under the action of moving loads. Fryba (Fryba 1990) made an important contribution to this by using Fourier transformation technique. In the study of Rafloyiamis (Raftoyiannis and Michaltsos 2008), the problem of an infinite mean on elastic foundation under the action of moving loads, exploiting the existence of the so-called quasistationary sate was presented.

A model involving dynamical analysis of vibration of thick rectangular plate under the action of moving concentrated load was taken up by Oni (Oni 2001). This plate model takes into consideration the effect of the rotatory inertia correction factor which was neglected in the non-Mindling model. The analysis was carried out for both simply supported and simple-clamped end conditions. It was found that when consideration is given to rotatory inertia correction factor, the response amplitude reduces. Celep and Turhan (Celep and Turhan 1990) analyzed the axisymmetric vibrations of a circular plate subjected to concentrated dynamic head at its centre.

Very recently, Mamandi et al (Mamandi et al 2015) studied the nonlinear dynamical behavior of a rectangular plate travelled by a moving mass as well as an equivalent concentrated force with non-constant velocity. They used Galerkin’s method to transform the governing equation of motion into a set of three coupled non-linear ODE which was solved in a semi-analytical way to get the dynamical response of the plate.

It is noted that the effect of a foundation can be modeled by various approaches to the plate problems. The simplest model presented for the elastic foundation is the Winkler
foundation model (WFM). This is the most rudimentary mechanical subgrade model which assumes that the shear resistance of the foundation is ignorable compared to the shear capacity of the foundation, and models the foundation as set of independent springs.

The vibration of rectangular plate resting on a non-uniform elastic Winkler foundation is considered by Lee and Lin (1993), where the Levy solution method and the Green’s function where employed in their study. Mofid and Noroozi (2009) considered an elastic plate on a modified Winkler foundation. The Kirchhoff theory is assumed for the plate and the Winkler coefficient is assumed to have variations versus position with the functionality of the domain along with the plate span.

In this present work, an elastic rectangular plate resisting on constant elastic subgrade and traversed by moving distributed load at constant velocity is considered. The coupled forth order partial differential equation (PDE) is decoupled by the method of separation of variable to a set of second order ordinary differential equation (ODE) and the dynamic influence of the vital parameters analyzed.

2. Problem Formulation

Consider a rectangular plate schematic shown below.

Fig.1 A typical rectangular plate

The governing equation of motion of the rectangular plate for the transverse displacement \( V(x, y, t) \) in \( z \) direction is given by

\[
D\nabla^4 W(x, y, t) + \mu_0 \frac{\partial^2}{\partial t^2} W(x, y, t) + kW(x, y, t) = \mu_0 R\frac{\partial^2}{\partial t^2} W(x, y, t) + P(x, y, t) \tag{2.1}
\]

where

\[
D = \frac{Eh^3}{12(1-\mu^2)} \tag{2.2}
\]

and it is called the flexural rigidity of the plate, \( h \) is the thickness of the plate, \( E \) is the Young’s modulus, \( \mu \) is poisson’s ratio, \( \rho_0 \) is the mass density per unit area of the plate. \( \nabla^2 \) is called Laplacian operator and defined as \( \nabla^4 = \nabla^2 \nabla^2 \) and \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \).

The simplest model which performs well in many areas of structural designs is the Winkler elastic foundation. The model assumes that the shear resistance of the foundation is disregarded compared to the shear capacity of the foundation, and models the foundation as a set of independent springs. The model experiences no lateral interaction between the springs. This is given by (Mofid and Noroozi 2009).

\[
KW(x, y, t) = K_0 W(x, y, t) \tag{2.3}
\]
In this work, the effect of the mass of the moving load on the response of the plate is taken into consideration and the load is expressed as a function of two spatial coordinate \( x \) and \( y \) in the form,

\[
P(x, y, t) = P_w(x, y, t)
\]

where

\[
P_w(x, y, t) = \sum_{i=1}^{\infty} M_i g H(x - c_i t) H(y - s)
\]

where

\[
H(\cdot) \text{ is the Heaviside function, } \nabla^* \text{ is substantive acceleration operator and } g \text{ is the acceleration due to gravity, } c_i \text{ is the constant velocities of the mass moving along straight line } y = s.
\]

\[
\Delta^* = \frac{\partial^2}{\partial t^2} + 2c \frac{\partial^2}{\partial x \partial t} + c^2 \frac{\partial^2}{\partial x^2}
\]

Substituting equations (2.3) to (2.6) into equation (2.1), one obtains

\[
D \nabla^4 W(x, y, t) + \mu_n \frac{\partial^2}{\partial t^2} W(x, y, t) = \mu_n \Omega^2 \left( \frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial x^2 \partial y^2} \right) W(x, y, t)
\]

\[
- K_n W(x, y, t) + \sum_{i=1}^{\infty} M_i g H(x - ct) H(y - s) - M_i \left( \frac{\partial^2}{\partial t^2} + 2c \frac{\partial^2}{\partial x \partial t} + c^2 \frac{\partial^2}{\partial x^2} \right) W(x, y, t) H(x - ct) H(y - s)
\]

To transform equation (2.7) into ordinary differential equation, the method of separation of variable is employed. This method requires that the deflection be written as

\[
\sum_{n=1}^{\infty} q_n(x, y) \phi_n(t)
\]

where \( q_n(x, y) \) are known as eigenfunctions of the orthotropic rectangular simply supported plate with the same boundary condition on Winkler elastic foundation and \( q_n \) have the form

\[
\nabla^2 q_n - w_n^4 q_n = 0
\]

where

\[
w_n^4 = \frac{\Omega_n^2 \mu_n}{D}
\]

and \( \Omega_n, n = 1, 2, 3, \ldots \) are natural frequencies of the dynamical system and \( \phi_n(t) \) are amplitude functions to be determined.

The substitution of equation (2.8) into equation (2.7) and after arrangements yield,

\[
\sum_{n=1}^{\infty} \left[ F_{n,xx}(x, y) \Phi_{n,n}(t) + q_{n,yy}(x, y) \Phi_{n,n}(t) \right] - \frac{k_n}{\mu_n} q_n(x, y) \Phi_{n,n}(t)
\]
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\[ + \sum_{i=1}^{N} \frac{M_{x}}{\mu} H(x - ct)H(y - s) - \frac{M_{y}}{\mu} \left( q_{n}(x, y)\Phi_{n,xx}(t) + 2c q_{n,xy}(x, y)\Phi_{n,xy}(t) \right) \]  

\[ + c^{2} q_{n,xx}(x, y)\Phi_{n}(t) \right] H(x - ct)(y - s) \} = \sum_{n=1}^{\infty} q_{n}(x, y)\Phi_{n}(t) \]  

(2.11)

Multiply equation (2.11) by \( q_{m}(x, y) \), integrate on an area \( A \) of the plate and considering the orthogonality of \( q_{n}(x, y) \) and after arrangements, one obtains

\[ \Phi_{n,xx}(t) + \frac{D_{n}}{\mu} \Phi_{n}(t) = F_{n} \sum_{i=1}^{\infty} \left[ q_{r,xx}(x, y)q_{n}(x, y)\Phi_{r,xx}(t)R_{o} + q_{r,yy}(x, y)q_{m}(x, y)\phi_{r,yy}(t)R_{o} \right] \]  

\[ - \frac{k_{0}}{\mu_{o}} q_{m}(x, y)\Phi_{n}(t)R_{o} + \sum_{i=1}^{\infty} \left[ \frac{M_{x}}{\mu_{o}} q_{m,xx}(x, y)H(x - ct)H(y - s) - \frac{MR_{o}}{\mu_{o}} q_{m,yy}(x, y)\Phi_{r,yy}(t) \right. \]  

\[ \left. + 2c q_{r,xx}(x, y)q_{m}(x, y)\Phi_{r}(t)R_{o} + c^{2} q_{r,xx}(x, y)q_{m}(x, y)\Phi_{r}(t) \right] H(x - ct)H(y - s)R_{o} \} \]  

(2.12a)

where

\[ F_{n} = \frac{1}{F_{o}} \]  

(2.12b)

Expressing the Heaviside functions in equation (2.12a) as Fourier series (13), for the two-dimensional structure, one obtains

\[ H(x - ct) = \frac{x}{L_{x}} + \frac{2}{m\pi} \sum_{m=1}^{\infty} \sin \frac{m\pi x}{L_{x}} \cos \frac{m\pi ct}{L_{x}} + C_{o} \]  

(2.13)

and

\[ H(y - s) = \frac{y}{L_{y}} + \frac{2}{k\pi} \sum_{k=1}^{\infty} \sin \frac{k\pi x}{L_{y}} \cos \frac{k\pi ct}{L_{y}} + C_{o} \]  

(2.14)

Defining the beam functions in the directions of \( x \) and \( y \) axis respectively as

\[ q_{m}(x, y) = \phi_{i}(x)\phi_{j}(y) \]  

(2.14a)

then

\[ \phi_{i}(x) = \sin \frac{\beta_{i}x}{L_{x}} + A_{i}\cos \frac{\beta_{i}x}{L_{x}} + B_{i}\sinh \frac{\beta_{i}x}{L_{x}} + C_{i}\cosh \frac{\beta_{i}x}{L_{x}} \]  

(2.14b)

\[ \phi_{j}(y) = \sin \frac{\beta_{j}y}{L_{y}} + A_{j}\cos \frac{\beta_{j}y}{L_{y}} + B_{j}\sinh \frac{\beta_{j}y}{L_{y}} + C_{j}\cosh \frac{\beta_{j}y}{L_{y}} \]  

(2.14c)

where \( A_{i}, A_{j}, \ldots, C_{i}, C_{j} \) are constants to be determined and \( B_{i}'s \) and \( B_{j}'s \) are mode frequencies. Substituting equations (2.13) into (2.14) into equation (2.12), and with further arrangements, one obtains

\[ \Phi_{n}(t) + \beta_{n}^{2}\Phi_{n}(t) - F_{n} \sum_{i=1}^{\infty} \left[ F_{n}A_{i}\Phi_{n}(t) - F_{n}A_{2}\Phi_{r}(t) \right] - \gamma \] \[ + \frac{2L_{x}}{k\pi} \sum_{k=1}^{\infty} \cos \frac{k\pi sL_{x}L_{y}}{L_{y}} A_{i}^{3}(k) \]  

\[ + c_{o}^{3} L_{y} A_{i}^{3} + \frac{2L_{x}}{m\pi} \sum_{k=1}^{\infty} \cos \frac{m\pi tsL_{x}L_{y}}{L_{y}} A_{i}^{3}(m,k) + \frac{4L_{x}}{m\pi^{2}k} \sum_{m=1}^{\infty} \cos \frac{m\pi ct}{L_{x}} \cos \frac{k\pi s}{L_{y}} A_{i}^{3}(m,k) \]  

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\[
\begin{align*}
+ 2e_o L_s \sum_{k=1}^{\infty} \frac{\cos \pi k \pi t}{L_x} A_0^k (m) + c_o L_s A_1^k + 2e_o L_s \sum_{k=1}^{\infty} \frac{\cos k \pi s}{L_y} A_1^k (k) + c_e^2 L_s L_y A_2^k \right) \Phi_y (t) \\
+ 2c A_1^1 + 4c L_s \sum_{k=1}^{\infty} \frac{\cos k \pi s}{L_y} A_1^1 (k) + 2c c_o L_s A_2^1 + 4c L_s \sum_{k=1}^{\infty} \frac{\cos m \pi c t}{L_x} A_1^1 (m) \\
+ \frac{8c}{m \pi} \sum_{k=1}^{\infty} \frac{\cos m \pi c t \cos k \pi s}{L_y} A_2^1 (m, k) + 4c L_s \sum_{k=1}^{\infty} \frac{\cos m \pi c t}{L_x} A_2^1 (m) + 2c c_o L_s p_s^2
\end{align*}
\]

\[
c^2 c_o L_s A_2^3 + \frac{2c^2 L_s}{m \pi} \sum_{k=1}^{\infty} \frac{\cos m \pi c t}{L_x} A_3^3 (m) + 4c^2 L_s \sum_{k=1}^{\infty} \frac{\cos m \pi c t \cos k \pi s}{L_y} A_3^3 (m, k)
\]

\[
+ \frac{2c^2 L_s}{m \pi} \sum_{k=1}^{\infty} \frac{\cos m \pi c t}{L_x} A_3^3 (m) + c^2 c_o L_s A_2^3 + \frac{2c^2 c_o L_s}{k \pi} \sum_{k=1}^{\infty} \frac{\cos k \pi s}{L_y} A_2^3 (k)
\]

\[
= \beta' \left[ L_x \cos \beta + A \sin \beta + B \cosh \beta + C \sinh \beta + \frac{\cos \beta ct}{L_x} - \frac{A \sin \beta ct}{L_x} - \frac{B \cosh \beta ct}{L_x} \right] \\
- \frac{C \sinh \beta ct}{L_x} \right) \frac{L_y}{j} \left[ - \cos \beta + A \sin \beta + B \cosh \beta + C \sinh \beta + \frac{\cos \beta s}{L_y} - \frac{A \sin \beta s}{L_y} \right] \\
- \frac{B \cosh \beta s}{L_y} - \frac{C \sinh \beta s}{L_y} \right] \right)
\]

(2.15)

where

\[
B = \frac{Dw}{\mu}, \quad F = \frac{1}{\rho}, \quad \beta' = \frac{M_s}{\mu}
\]

(2.16)

Equation (2.14) is now the transformed equation of our dynamical system under moving distributed loads. It could be solved for different boundary conditions.

3. Solution Technique

The coupled second order ordinary differential equation (2.16) (where all terms are considered) is solved using approximate analytical solution given by Struble’s asymptotic technique. To this end, equation (2.16) is re written as

\[
\frac{d}{dt} \Phi_y (t) + \Omega_j \Phi_y (t) + F_n \sum_{i=1}^{\infty} e \left[ \left( A_1^i + 2L_s \sum_{i=1}^{\infty} \frac{\cos k \pi s L_x L_y}{L_y} A_1^i (k) + c_o L_s A_2^i \right) \right]
\]

\[
+ 2L_s \sum_{i=1}^{\infty} \frac{\cos m \pi c t L_x L_y}{L_y} A_2^i (m, k) + \frac{4L_s}{m \pi} k \sum_{i=1}^{\infty} \frac{\cos m \pi c t \cos k \pi s}{L_y} A_3^i (m, k)
\]

(2.16)
\[
\left\{ \begin{array}{l}
2cA^1_i + \frac{4cL_x}{k\pi} \sum_{k=1}^{\infty} \cos k\pi sL_xL_y \cdot A^3_i(k) + 2cc_o L_yA^3_i + \frac{4cL_y}{m\pi} \sum_{m=1}^{\infty} \cos m\pi ctL_yL_x \cdot A^3_i(m) \\
+ \frac{8cL_xL_y}{m\pi} \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} \cos m\pi ctL_yL_x \cdot A^3_i(m,k) + \frac{4cL_yL_x}{k\pi} \sum_{k=1}^{\infty} \cos k\pi sL_yL_x \cdot A^6_i(m) + 2cc_o L_yL_xL_y^2 \phi_i(t)
\end{array} \right. \\
+ \frac{4cc_o L_xL_y}{k\pi} \cos k\pi sL_yL_x \cdot A^6_i(k) + \frac{2cc_o L_yL_x}{k\pi} \sum_{k=1}^{\infty} \cos k\pi sL_yL_x \cdot A^6_i(k)
\]

\[
c^2c_o L_xA^3_i + \frac{2c^2c_o L_xL_y}{m\pi} \sum_{m=1}^{\infty} \cos m\pi ctL_yL_x \cdot A^6_i(m) + \frac{4c^2L_xL_y}{mk\pi^2} \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} \cos m\pi ctL_yL_x \cdot A^6_i(m,k)
\]

\[
+ \frac{2c^2c_o L_xL_y}{m\pi} \sum_{m=1}^{\infty} \cos m\pi ctL_yL_x \cdot A^6_i(m,k) + \frac{4c^2c_o L_yL_x}{m\pi} \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} \cos m\pi ctL_yL_x \cdot A^6_i(m,k)
\]

\[
= \beta^i \left[ \frac{L_x}{t} \left( \cos \beta_i + A_i \sin \beta_i + B_i \cosh \beta_i + C_i \sinh \beta_i + \frac{\cos \beta_i \text{ct}}{L_y} - \frac{A_i \sin \beta_i \text{ct}}{L_y} - \frac{B_i \cosh \beta_i}{L_y} - \frac{C_i \sinh \beta_i}{L_y} \right) \right]
\]

where
\[
\Omega_j = \Omega \left( 1 + \frac{\varepsilon_o G_m}{2} \right) \\
\varepsilon = \frac{M}{\mu L_xL_y}
\]

Equation (3.1) is arranged to take the form
\[
\frac{d^2}{dt^2} \Phi_j(t) + \frac{\varepsilon G_{2\alpha}(t)}{1 + \varepsilon G_{1\alpha}(t)} \frac{d}{dt} \Phi_j(t) + \frac{\Omega_j^2 + G_{3\alpha}(t)\Phi_j(t)}{1 + \varepsilon G_{1\alpha}(t)} \\
+ \frac{\varepsilon}{1 + \varepsilon G_{1\alpha}(t)} \sum_{r=1}^{\infty} G_{1\alpha}(t) \frac{d^2}{dt^2} \Phi_j(t) + G_{2\alpha}(t) \frac{d}{dt} \Phi_j(t) + G_{3\alpha}(t)\Phi_j(t)
\]

\[
= \frac{\varepsilon_g}{(1 + \varepsilon G_{1\alpha}(t))} \Phi_j(t) F_n \phi_j(s)
\]
When the homogenous part of the equation (3.4) is considered, one obtains a modified frequency corresponding to the frequency of the free system due to the presence of the moving mass (Oni and Ogunyebi 2008).

Therefore, the solution of equation (2.13) can be written as

\[
\Phi_n(t) = \eta_m \cos \left[ \Omega_f t - \phi_m \right]
\]

(3.8)

where \( \eta_m \) and \( \phi_m \) are constants.

The asymptotic solution of the homogenous part of the equation (3.4) is given

\[
\Phi_n(t) = B_n(t) \cos \left[ \Omega_f t - \phi_n(t) \right] + \mu_n \Phi_1(t) + O(\mu_n^3)
\]

(3.9)

where \( B_n(t) \) and \( \phi_n(t) \) are slowly varying function of time.

When equation (3.9) and its derivatives are substituted into equation (3.4) and noting the terms below which do not contribute to the variational equation.

\[
\cos \frac{k \pi s}{L_y} \sin \left( \Omega_f (t) - \phi(t) \right)
\]

\[
\cos \frac{m \pi x}{L_y} \cos \left( \Omega_f (t) - \phi(t) \right)
\]

\[
\cos \frac{m \pi x}{L_y} \sin \left( \Omega_f (t) - \phi(t) \right)
\]
\[
\cos \frac{k \pi s}{L_y} \cos \left( \Omega_f (t) - \phi(t) \right)
\]

(3.10)

One obtains for \( B_n(t) \)

\[
-2 B_n(t) - 2 c \mu_0 A_1^2 B_n(t) - 2 \left( c \mu_0 L_y L_z A_1^2 \right) B_n(t)
\]

\[
-2 c \mu_0 L_y L_z A_1^2 B_n(t) - 2 \left( c \mu_0 L_y L_z A_1^2 \right) B_n(t)
\]

(3.11)

for which the solution gives

\[
B_n(t) = C_n^{\alpha} e^{-\gamma_n t}
\]

(3.12)

where \( C_n^{\alpha} \) is a constant and

\[
\gamma_n = \frac{c \mu_0 A_1^2 - c \mu_0 L_y A_1^2 - c \mu_0 L_z A_1^2 - c \mu_0 L_y L_z A_1^2}{\Omega_f}
\]

(3.13)

Then for \( \phi_{mn}(t) \),

\[
2 \Omega_f \dot{\phi}_{mn}(t) + c^2 \mu_0 A_1^2 + c^2 c_0 \mu_0 A_1^2 + c^2 c_0 L_y \mu_0 A_1^2 + c^2 c_0 L_z \mu_0 A_1^2
\]

\[
- \mu_0 A_1^2 \Omega_f^2 - c_0 L_y \mu_0 A_1^2 \Omega_f^2 - c_0 L_z \mu_0 A_1^2 \Omega_f^2
\]

(3.14)

and

\[
\phi_{mn}(t) = \frac{\mu_0}{2} \left\{ \Omega_f \left( A_1^2 + c^2 c_0 L_y A_1^2 + A_1^2 \frac{c_0}{L_y} - c_1^2 L_y \right) \left( \frac{c^2 A_1^2 + c^2 c_0 L_y A_1^2 + c^2 c_0^2 L_y A_1^2}{\Omega_f} \right) \right\} + \gamma_{mn}
\]

(3.15)

where \( \gamma_{mn} \) is a constant.

Therefore when the mass effect of the particle is considered, the first approximation to the homogenous system is given as

\[
\Phi_n(t) = \eta_m e^{-\gamma_n t} \cos \left( \Omega_f t - \phi_{mn} \right)
\]

(3.16)

where

\[
\Omega_f = \left[ 1 - \frac{\mu_0}{2} \left\{ A_1^2 + c^2 c_0 L_y A_1^2 + A_1^2 \frac{c_0}{L_y} - c_1^2 L_y \right\} - \frac{\mu_0}{2 \Omega_f^2} \left( c^2 A_1^2 + c^2 c_0 L_y A_1^2 + c^2 c_0^2 L_y A_1^2 \right) \right]
\]

(3.17)

which is the modified frequency corresponding to the presence of the moving mass.

Thus equation (3.4) becomes

\[
\frac{d^2}{dt^2} \Phi_n(t) + \Omega_f^2 \Phi_n(t) = \frac{g_x L_y L_z}{F_n} \left( p_m + \eta_v(t) \right)
\]

(3.18)

To obtain the solution to equation (3.18), it is subjected to Laplace transform and convolution theory and when inverted yields
where

\[ p_m = -\cos \phi_m - A_m \sin \phi_m + B_m \cosh \phi_m + C_m \sinh \phi_m \]  

and

\[ \eta_m(t) = \cos \alpha \omega t - A_m \sin \omega t - B_m \cosh \alpha \omega t + C_m \sinh \alpha \omega t \]  

Equation (3.19) represents the dynamic response of a moving distributed mass of orthotropic rectangular plate on constant Winkler elastic subgrade at uniform speed.

The corresponding moving force solution is given by

\[
W(x, y, t) = \frac{\varepsilon_x L_x L_y}{F_n} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} P_m \left[ -\frac{P_m \sin \Omega_{mf} t}{\Omega_{mf}^2 - \alpha_m^2} - \sin \alpha_m t + \sin \Omega_{mf} t \right]
- \frac{\cos \alpha_m t + \cos \Omega_{mf} t}{\Omega_{mf}^2 - \alpha_m^2} - \frac{B_m 2\Omega_{mf}^2 \alpha_m \sin 2\Omega_{mf} t \cosh \alpha_m t}{\Omega_{mf}^2 - \alpha_m^4} - \frac{B_m \alpha_m^2 \cos 2\Omega_{mf} t \sin \alpha_m t}{\Omega_{mf}^4 - \alpha_m^4}
+ \frac{B_m \alpha_m \sin 2\Omega_{mf} t (\alpha_m^2 - \Omega_{mf}^2)}{\Omega_{mf}^4 - \alpha_m^4}
+ \frac{C_m 2\Omega_{mf}^2 \alpha_m \sin 2\Omega_{mf} t \sinh \alpha_m t}{\Omega_{mf}^4 - \alpha_m^4}
+ \frac{C_m \alpha_m^2 \cos 2\Omega_{mf} t \cosh \alpha_m t}{\Omega_{mf}^4 - \alpha_m^4}
+ C_m \left( \frac{\alpha_m^2 - \Omega_{mf}^2}{\Omega_{mf}^4 - \alpha_m^4} \right) \right]
\]  

(3.19)
\[
\times \left[ \left( \frac{\alpha_m x}{L_x} + A_m \cos \frac{\alpha_m x}{L_x} + B_m \sinh \frac{\alpha_m x}{L_x} + C_m \cosh \frac{\alpha_m x}{L_x} \right) \right.
\]
\[
\cdot \left( \frac{\alpha_m y}{L_y} - A_m \cos \frac{\alpha_m y}{L_y} - B_m \sinh \frac{\alpha_m y}{L_y} + C_m \cosh \frac{\alpha_m y}{L_y} \right) \right]
\]

(3.19)

4. Comments on closed form solution

In the dynamic analysis of orthotropic plate resting on Winkler elastic foundation, it is necessary to study the phenomenon of resonance especially for the undamped system under consideration.

Equation (3.22) clearly shows that the moving force at uniform velocity reaches a state of resonance whenever

\[
\Omega_{mf} = \frac{m \pi c t}{L_x}
\]

(4.1)

while equation (3.91) shows that the same orthotropic plate under the action of moving mass experience resonance whenever

\[
\Omega_f = \frac{m \pi c t}{L_x}
\]

(4.2)

where

\[
\Omega_f = \left[ 1 - \frac{\mu_c}{2} \left\{ A_1 + A_3 c_s L_x + A_3 c_s^2 L_x L_y \right\} - \frac{\mu_s}{2 \Omega_f} \left\{ c^2 A_1 + c^2 c_s L_x A_3 + c^2 c_s^2 L_x L_y A_3 \right\} \right]
\]

(4.3)

Equation (4.2) and (4.3) simply imply

\[
\Omega_f = \left[ 1 - \frac{\mu_c}{2} \left\{ A_1 + A_3 c_s L_x + A_3 c_s^2 L_x L_y \right\} - \frac{\mu_s}{2 \Omega_f} \left\{ c^2 A_1 + c^2 c_s L_x A_3 + c^2 c_s^2 L_x L_y A_3 \right\} \right] = \frac{m \pi c t}{L_x}
\]

(4.4)

Thus, it can be deduced from equation (4.4) that, the critical speed (and the natural frequency) for the system traversed by a moving distributed mass is smaller than that of the system traversed by a moving distributed force. Thus, resonance is reached earlier in the moving distributed mass system than in the moving distributed force system.

5. Conclusion

In this work, simplified analytical solutions for the dynamic response of orthotropic plate under moving distributed loads has been considered. First, the coupled first order partial differential equation describing the vibrating structure is decoupled by Struble’s asystomic technique. Then, the analytical solutions for the dynamic deflections for moving distributed mass (when the inertia term is retained) is obtained and solutions for the case when the inertial term is set to zero called moving force solution is also obtained. Finally, resonance condition is established as a guide to civil and structural engineers for appropriate precautions. For practical purpose, the mathematical procedures developed are applicable to all classical and non-classical boundary conditions.

References


